Advances in Chemical Engineering

Chapter 3

Mathematical Modelling of Rotating Disc Electrodes and Nonlinear Diffusion Equations

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Abstract

The Rotating Disc Electrode (RDE) technique has proved to be of considerable use in the study of electrode processes. In this chapter, mathematical models for a rotating disc electrode for the steady and transient states are discussed. Rotating disc electrodes can be modeled with linear and non-linear convection differential equations of EC', EC, Disp, and ECE reactions mechanism. The exact analytical solution of the non-linear convective diffusion problem is possible only for relatively simple cases. But for more complex cases, incorporating homogeneous reaction as well as heterogeneous charge transfer, the usual approach has not been used to find the solution of the differential equations. In this chapter, the recent modeling developments (analytical solution) of the chronoamperometric and

potentiometric current produced in a rotating disk electrode from all the electrochemical reactions are reviewed.

Keywords: Mathematical Modelling; Rotating Disc Electrode; Convection-Diffusion Equation; Nonlinear Equations; Analytical Solution; Numerical Solution.

1. Introduction

Rotating Disc Electrodes (RDEs) have found considerable application in the study of those electrode reactions involving electron transfers coupled to homogeneous chemical reactions. The current density measured from the rotating electrode is contributed by both the current densities of electrode electron transfer reaction and the reactant diffusion. It is essential to research and understand both the theories of the electrode electron-transfer reaction and the reactant diffusion in order to obtain the kinetic parameters of these two processes and their reaction mechanisms, based on the experiment's results.

The central part of the RDE theory and technique is the convection of the electrolyte solution. According to the convection of the solution, the reactant in the solution should flow at the same transport rate. Let’s first consider the situation where electrolyte solution flows upward from the bottom of the electrode edge with a direction parallel to the electrode surface to see how the diffusion convection layer can be formed and what is its mathematic expression. The flexibility of microelectrodes to interrogate fast electrochemical reactions has resulted in a wide range of analytical, semi-analytical, and numerical methods to solve many predominantly first-order mechanisms (E, CE, EC, EC’, ECE, DISP1, DISP2, and EC2E).

2. Nonlinear Equation in Rotating Disc Electrode

The system of second-order non-linear equations in rotating disk electrodes and their studies arises in various contexts such as electrochemical cell [1] and flow and heat transfer process in fluids [2,3] among others. Von Kármán swirling viscous flow [4] is a famous classical problem in fluid mechanics. The computational tools developed to simulate the setup also constitute one of the most rigorously studied systems in electrochemical engineering [5-8]. Hydrodynamic electrochemistry at rotating disc electrodes has been widely used to study electrode kinetics and mechanism of different kinds of reactions [9-11].

A chemical reaction couples two electrode reactions if the product of the first electrode reaction is the reactant of a chemical reaction, and the product of the latter is a reactant of the second electrode reaction [12-16]. For steady-state conditions, Levich [17] obtained the analytical expression for limiting the current of the rotating electrode under the assumption of infinite Schmidt numbers (Sc). Compton et al. [18] obtained the chronoamperometry current for ECE, DISP1, DISP2, EC, and CE reaction by solving the convective diffusion equation using the Hales method. Lin et al. [19] derived the catalytic current at a rotating disk electrode using

the perturbation method. Bartlett et al. [20] derived the approximate analytical expression of flow at a rotating disc electrode for ECE reactions for various limiting cases.

Chitra et al. [21] derived the approximate analytical expressions for the velocity component from small and long-distance expressions using the Padé approximation method for all values of dimensionless distance. Saravanakumar et al. [22] obtained the non-steady state current at a rotating disk electrode for all time by solving the convection-diffusion equation analytically. Jansi Rani et al. [23] reported current at a rotating disk electrode under transient and steady-state conditions using the homotopy perturbation method.

Recently Visuvasam et al. [24] derived an analytical expression of the current generated from the electrochemical reaction in a porous rotating disk electrode (PRDE). Saravanakumar et al. [25] obtained the analytical expression of concentrations and current for a rotating disc electrode for E, EC' and ECE reactions for all values of parameters. Kirthiga et al. [26] developed the theoretical analysis which describes transport and kinetics at electrodes, which have been chemically modified with highly dispersed meshes of single-wall carbon nanotubes.

Saravanakumar et al. [27] solved the nonlinear convective migration diffusion equation in the rotating disc electrode. A simple closed-form analytical expression for the concentration of a three-ion system is derived under the assumption that all ions have the same diffusivity of electrode processes. He recently solved the one-dimensional convection-diffusion equation and its fractional modification for E reaction arising in rotating disk electrodes [28]. Diard and Montella [29] obtained the steady-state concentration of species near a uniformly accessible rotating disk electrode, using both symbolic and numerical methods. Visuvasam et al. [30] obtained the analytical expression for concentration profile and current at the rotating disc electrode.

3. Analytical Solutions of Rotating Disc Electrodes

The concentration/current at a rotating disc electrode is controlled by diffusion, convection, and migration. Non-linear phenomena play a crucial role in physical chemistry and biology (heat and mass transfer, filtration of liquids, diffusion in chemical reactions, etc.). In the past several decades, many authors mainly paid attention to the resolution of non-linear equations by using various analytical and numerical methods, such as the variational iteration method(VIM) [31-34], the homotopy perturbation method (HPM) [35-38] and the Adomian decomposition method (ADM)[39-41], Hales method [18], Taylor series method [28,30], Pade approximation method [21,30], exp-function method [42], Hyperbolic function method [30].

4. Numerical Solutions of Rotating Disc Electrodes

Electrochemical simulations are one particular approach to understand the processes

at electrodes [43-46]. Ming et al. [3] solved the system of highly non-linear differential equations using the multi-shooting methods. Diard et al. [29] used the numerical methods to obtain the steady-state concentration of species near a uniformly accessible rotating disk electrode. White et al. [47] solved the problem numerically by Newman technique [48]. Mathematical demonstration using Mathematica software for von Kármán swirling flow of RDEis created by Higgins and Binous [49].

Bikash Sahool et al. [50] adopting the direct multiple shooting method for the solutions of a coupled and non-linear system of differential equations, arising due to the steady Kármán flow and heat transfer of a viscous fluid in a porous medium. Porous enzymatic electrodes following DET [51] and MET [52] mechanisms have also been simulated. The models pointed out that the major limitation was the mass transfer limitation. Theoretical and numerical simulations of diffusion and kinetics in amperometric immobilized enzyme electrodes for redox mediator entrapped within the film using the relaxation method was investigated by Bartlett et al. [53].

5. Analytical Expressions of Concentrations and Current

The recent contributions to the analytical expression of concentration and current for rotating disc electrodes for various mechanisms are given the Table-1.

[29]	Chronoamperometric. (steady-state)	$A + e^- \rightarrow B$	$\frac{d^2U(x)}{dx^2} - \delta \quad H(x) \quad \frac{dU(x)}{dx} = 0$ B.C. $U(0) = 1, \; U(\infty) = 0$	$U(x) = \frac{\Gamma(1/3, x^3)}{\Gamma(1/3)}$ $\frac{J_{lim}(\delta)}{J_{lim}(\infty)} = 1 - \frac{0.2980062}{\delta^{1/3}} - \frac{0.056735}{\delta^{2/3}} - \frac{0.010163}{\delta} - \frac{0.003026}{\delta^{4/3}} + O(\delta^{-5/3})$
[30]	Chronoamperometric. (steady-state)	$B + C \overset{k}{\rightarrow} D$ $D \pm e^- \leftrightarrows E$	$\frac{d^2v(\chi)}{d\chi^2} - \kappa\gamma u(\chi)v(\chi) = 0$ $\frac{d^2u(\chi)}{d\chi^2} - \kappa\, u(\chi)v(\chi) = 0$ $\frac{d^2w(\chi)}{d\chi^2} + \kappa u(\chi)v(\chi) = 0$ B.C. $\frac{du}{d\chi} = -1, \; \frac{dv}{d\chi} = 0, \; w = 0 \quad \text{at } \chi = 0$ $u=0, v=1, w=0 \quad \text{at } \chi=1$	$u(\chi) = \frac{1}{\gamma}\left[\frac{\cosh(l\chi)}{\cosh(l)} - 1\right] + (1-\chi)$ $v(\chi) = \frac{\cosh(l\chi)}{\cosh(l)}$ $w(\chi) = \frac{1}{\gamma}\left[\chi - \frac{\cosh(l\chi)}{\cosh(l)} + (1-\chi)\frac{1}{\cosh(l)}\right]$ $\psi = \frac{i_L}{nFAD[A]_\infty} = \frac{1}{\delta_{diff}}\left(1 + \left(\frac{\cosh(l)-1}{\gamma\ \cosh(l)}\right)\right)$ where $\frac{l^2}{\kappa} - \cosh(0.3l)\,\mathrm{sech}(l) + 1 - 0.7\gamma = 0$

[26]	Chronoamperometric. (steady-state)	$A+e^{-}\rightarrow B$	$\frac{d^2u}{d\chi^2}-\frac{\gamma_S u}{1+(\alpha+\kappa)u}=0$ $\frac{d^2v}{d\chi^2}+\frac{\gamma_M u}{1+(\alpha+\kappa)u}=0$ B.C $\chi=0,\left(\frac{du}{d\chi}\right)=0$ $v=v_0=\zeta^{-1}\left.\frac{dv}{d\chi}\right\|_{\chi=0}$ $\chi=1,\ u=1$	$u(\chi)=\frac{\cosh\ m\ \chi}{\cosh\ m}$ $v(\chi)=\frac{A}{m^2}\left[\cosh m-\cosh m\chi+\frac{\zeta(1-\cosh m)}{\zeta+1}(1-\chi)\right]$ $\psi_S=m\ \tanh m$ $\psi_\Sigma=\frac{\gamma_M}{\gamma_S}\psi_S+\left(\frac{dv}{d\chi}\right)_{\chi=1}$ Where $m=\sqrt{\frac{\gamma_S}{1+\alpha+\kappa}}$ $A=\frac{\gamma_M}{(1+\alpha+\kappa)\cosh m}$
[27]	Voltammetry (Steady-state)	$R-(z_P-z_R)$ $e^{-}\rightarrow P$	$\frac{d^2\mu}{dx^2}+\frac{z_i i}{2FD_i}\left[c_i\left(\mu-\frac{z_j z_k}{z_R z_C}\mu^b\right)\frac{1}{\mu^2}\frac{d\mu}{dx}+\frac{1}{\mu}\frac{d\mu}{dx}\right]$ $+\frac{K}{D_i}\sqrt{\frac{\omega^3 d}{\eta}}\frac{d\mu}{dx}=0$ B.C. $\frac{\mu}{\mu^b}=\frac{z_P}{z_R}\quad at\ x=0$ $\frac{\mu}{\mu^b}=1\quad at\ x\rightarrow\infty$	$\frac{\mu(\chi)}{\mu^b}=1+\frac{1}{(u_1(0))^2}\left(\frac{z_P}{z_R}-1\right)\left[u_1(0)u_1(\chi)+\lambda z_P\left\{u_1(0)u_2(\chi)-u_2(0)u_1(\chi)\right\}\right]$ Normalized current density $\lambda=\frac{1}{z_C-z_R}\left.\frac{d\rho}{d\chi}\right\|_{\chi=0}=\frac{1.1198\ (z_R-z_P)}{z_R(z_C-z_R)+0.5661 z_P(z_R-z_P)}$ The ratio of two steady state currents $\frac{i}{i_{levich}}=\Gamma\left(\frac{4}{3}\right)\frac{\lambda z_R(z_R-z_P)}{z_C-z_R}=\left[\frac{1}{1+\frac{0.5661\ z_P(z_R-z_P)}{z_R(z_C-z_R)}}\right]$ Where $i_{levich}=\frac{(z_P-z_R)Fc_R^b}{\Gamma\left(\frac{4}{3}\right)}\left[\frac{KD^2}{3}\sqrt{\frac{d}{\eta}}\right]^{1/3}\sqrt{\omega}$ $\chi=x\left(\frac{K}{3D_i}\sqrt{\frac{d}{\eta}}\right)^{1/3},\ \rho=\frac{\mu}{\mu^b}=\frac{\mu}{z_R(z_R-z_C)c_R^b},\lambda=\frac{i}{2F\mu^b\sqrt{\omega}}\left(\frac{3}{KD_i^2}\sqrt{\frac{\eta}{d}}\right)^{1/3}$
[28]	Chronoamperometric. (steady-state)	$A+e^{-}\rightarrow B$	$\frac{d^2a}{dx^2}+x^2\frac{d}{dx}=0$ $a(0)=0,\ a'(l)=1$	

	EC' reaction	$B \rightarrow C$ $C + e^- \rightarrow D$		
		$A \pm e^-_k \Leftrightarrow B$ $B + S \rightarrow A + Y$	$-D_A \frac{d^2A}{dx^2} - Cx^2 \frac{dA}{dx} - k_s B = 0$ $-D_B \frac{d^2B}{dx^2} - Cx^2 \frac{dB}{dx} + k_s B = 0$ $A(x=0) = 0,\ A(x=L) = A_\infty$ $B(x=0) = A_\infty,\ B(x=L) = 0$ $I_{EC'} = J = -\left.\frac{db}{d\xi}\right\|_{\xi=0}$	$a(\xi) = \frac{A(x)}{A_\infty} = \xi\left(\frac{k_{EC'}l}{3} + \frac{l^2}{12} + \frac{1}{l}\right) - \frac{k_{EC'}\xi^2}{2} + \frac{k_{EC'}\xi^3}{6l} - \frac{\xi^4}{12l}$ $= \left(\frac{1}{L} + \frac{k_{EC'}L}{3D_A} + \frac{0.51023\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}L^2}{12D_A}\right)x - \frac{k_{EC'}}{2D_A}x^2 + \frac{k_{EC'}}{6D_A L}x^3 - \frac{0.51023\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}}{12D_A L}x^4$ $b(\xi) = \frac{B(x)}{A_\infty} = 1 - \xi\left(\frac{k_{EC'}\mu l}{3} + \frac{\mu l^2}{12} + \frac{1}{l}\right) + \frac{k_{EC'}\mu\xi^2}{2} - \frac{k_{EC'}\mu\xi^3}{6l} + \frac{\mu\xi^4}{12l}$ $= 1 - \left(\frac{1}{L} + \frac{k_{EC'}L}{3D_B} + \frac{0.51023\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}L^2}{12D_B}\right)x + \frac{k_{EC'}}{2D_B}x^2 - \frac{k_{EC'}}{6D_B L}x^3 + \frac{0.51023\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}}{12D_B L}x^4$ The dimensionless current becomes $I_{EC'} = J = -\left.\frac{db}{d\xi}\right\|_{\xi=0}$ $= \frac{k_{EC'}\mu l}{3} + \frac{\mu l^2}{12} + \frac{1}{l} = \left(\frac{0.51023v^{-\frac{1}{2}}\omega^{\frac{3}{2}}}{D_A}\right)^{\frac{-1}{3}}\left(\frac{1}{L} + \frac{k_{EC'}L}{3D_B} + \frac{0.51023v^{-\frac{1}{2}}\omega^{\frac{3}{2}}L^2}{12D_B}\right)$ The limiting current is given by $\psi_{EC'\lim} = \frac{I_{EC'}}{I_{EC'\lim}} = 1 + \frac{4k_{EC'}\mu l^2}{\mu l^3 + 12} = 1 + \frac{4k_{EC'}L^2}{0.51023v^{-\frac{1}{2}}\omega^{\frac{3}{2}}L^3 + 12D_B}$ The steady state current response at an RDE for steady state voltammetry can be expressed $\psi_{SVEC'} = \frac{1}{1+e^{\eta}}\left(1 + \frac{4k_{EC'}L^2}{0.51023\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}L^3 + 12\,D_B}\right)$ where $\eta\left(=\frac{nF}{RT}(E - E^0)\right)$ is the potental.

[24]	Chronoamperometric. (steady-state)	$O + e \leftrightarrow R$	$(-\alpha Z^2 - \beta)\frac{dc}{dz} = \frac{D_f}{v}\frac{d^2c}{dz^2}$ $(-\gamma Z - \beta)\frac{d\hat{c}}{dz} = \frac{D_p}{v}\frac{d^2\hat{c}}{dz^2} - \frac{k_r}{\Omega}\hat{c}$ *B.C* $c = 1, \quad z \to \infty$ $\hat{c} = 0, \ z \to -\infty$ $c = \hat{c}, \qquad z = 0$ $\frac{dc}{dz} = \frac{D_p}{D_f}\frac{d\hat{c}}{dz}, \ z = 0$ where $\alpha = 0.51, \ \beta = 2kh(\Omega/v)^{3/2}, \ \gamma = 2k\Omega/v$	$c(y) = 1 + (m_1 + k_2)\,\Gamma\left(\frac{1}{3}, y^3\right) - B\,m_1\,\Gamma\left(\frac{2}{3}, y^3\right)$ $\hat{c}(y) = e^{\alpha y}\left[k_1 + l_0 + l_1 y + \frac{b}{4a}y^2\right]$ where $m_1 = -\frac{Dk_1\,a}{3}, \ k_1 = \frac{3}{3 + 2.6789\ Da}, \quad k_2 = \frac{-D[l_1 + l_o\,a]}{3} = \frac{l_0 + l_2}{2.6789},$ $b = -\frac{Gk_1\,a}{D}, \ l_0 = \frac{-3l_2 - 2.6789\ D\ l_1}{2.6789\ Da + 3}, \ m = -\frac{B}{D}\ k_1\,a,$ $l_1 = -\frac{b}{4a^2} + \frac{m}{2a}, l_2 = 1.3541\,Bm_1\,, \ a = \sqrt{\frac{K}{D}}$ The current becomes, $I\sqrt{\frac{K}{D}} = \frac{1 + 0.8930\sqrt{KD} + 0.4514B\sqrt{KD} - \frac{0.8930\,GD}{\sqrt{KD}} + 0.8930B - \frac{0.75\,G}{K} + \frac{0.5\,B}{\sqrt{KD}}}{(1 + 0.8930\sqrt{KD})^2}$
[25]	chronoamperometric and cyclic voltammetry (Steady state)	$A + e^- \to B$	$-D_A\frac{d^2A}{dx^2} + v_x\frac{dA}{dx} = 0, \ 0 < x < X$ $v_x = -Cx^2 \quad \text{and} \quad C = 0.51023\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}$ $A(x=0) = 0, \ A(x=L) = A_\infty$ $I_0 = J(a) = \left.\frac{da}{d\xi}\right\|_{\xi=0}$ $a = \frac{A}{A_\infty}, \ \xi = \left(\frac{C}{D_A}\right)^{\frac{1}{3}}x, \ l = \left(\frac{C}{D_A}\right)^{\frac{1}{3}}L$	$a(\xi) = \frac{A(x)}{A_\infty} = \frac{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \frac{\xi^3}{3}\right)}{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \frac{l^3}{3}\right)} = \frac{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \frac{0.170076667\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}x^3}{D_A}\right)}{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \frac{0.170076667\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}L^3}{D_A}\right)}$ $I_0 = J(a) = \frac{3^{\frac{2}{3}}}{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \frac{l^3}{3}\right)} = \frac{2.0801}{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \frac{0.170076667\,v^{-\frac{1}{2}}\omega^{\frac{3}{2}}L^3}{D_A}\right)}$ when $l = \infty$, the concentration and the current becomes $a(\xi) = \frac{A(x)}{A_\infty} = \frac{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \frac{\xi^3}{3}\right)}{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \infty\right)} = 0.37328\left[\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \frac{0.170076667\ v^{-\frac{1}{2}}\omega^{\frac{3}{2}}x^3}{D_A}\right)\right]$ $I_0 = J(a) = \frac{3^{\frac{2}{3}}}{\Gamma\left(\frac{1}{3}\right) - \Gamma\left(\frac{1}{3}, \infty\right)} = 0.7765$ where $a = \frac{A}{A_\infty}, \ \xi = \left(\frac{C}{D_A}\right)^{\frac{1}{3}}x, \ l = \left(\frac{C}{D_A}\right)^{\frac{1}{3}}L$

Ref	Experimental techniques	Enzymatic scheme	Nonlinear Diff. Eqns. with initial/ boundary conditions	Expressions for Concentration and current
[21]	steady-state laminar flow	$O + e \leftrightarrow R$	$\frac{dH}{d\zeta} + 2F = 0$ $\frac{d^2F}{d\zeta^2} - H\frac{dF}{d\zeta} - F^2 + G^2 = 0$ $\frac{d^2G}{d\zeta^2} - H\frac{dG}{d\zeta} - 2FG = 0$ $\frac{d^2H}{d\zeta^2} - H\frac{dH}{d\zeta} + \frac{dP}{d\zeta} = 0$ $F(\zeta = 0) = 0,\ \ G(\zeta = 0) = 1,\ \ H(\zeta = 0) = 0$ $F(\zeta \to \infty) = 0,\ \ G(\zeta \to \infty) = 0,\ P(\zeta \to \infty) = 0$	$\text{Velocity} = \frac{p_0 + p_1 x + p_2 x^2 + p_3 x^3 + p_4 x^4 + p_5 x^5}{1 + q_1 x + q_2 x^2 + q_3 x^3 + q_4 x^4 + q_5 x^5}$ The value of the constant p_0 to p_5 and q_1 to q_5 for angular velocity, axial velocity and radial velocity are given in Ref.[21].
[22]	Amperometric . (non-steady state)	$O + e \leftrightarrow R$	$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial z^2} + az^2\frac{\partial c}{\partial z}$ $c(z,0) = c_0, c(\infty,t) = c_0, c(0,t) = 0$ where $a = 0.51023\, \nu^{\frac{-1}{2}} \Omega^{\frac{3}{2}}$	$\frac{c}{c_0} = erf\left(\frac{\zeta}{2\sqrt{\tau}}\right) + erfc\left(\frac{\zeta}{2\sqrt{\tau}}\right)\left[\frac{\zeta^3}{24} + \frac{\zeta\tau}{4}\right] - \frac{\zeta^2\sqrt{\tau}}{4\sqrt{\pi}} e^{\frac{-\zeta^2}{4\tau}} + C_{N2}(\zeta,\tau)$ $j(\tau) = D\left(\frac{\partial c}{\partial z}\right)_{z=0} = \frac{1}{\sqrt{\pi\tau}} + \frac{\tau}{4} + 0.016666\, \tau^{5/2}$ Where $\tau = (Da^2)^{\frac{1}{3}} t;\quad \zeta = \left(\frac{a}{D}\right)^{\frac{1}{3}} z$
[23]	Chronoamperometric, Normal Pulse voltammetry and Steady- State voltammetry	$O + e \leftrightarrow R$	$\frac{\partial c_i}{\partial t} + v_z\frac{\partial c_i}{\partial z} = D_i\frac{\partial^2 c_i}{\partial z^2} \quad (i \equiv O,\ R)$ $v_z = -0.51023\ \nu^{-1/2}\Omega^{3/2} z^2 + \frac{1}{3}\nu^{-1}\Omega^2 z^3 + \ldots$ *B.C* $c_O(z,\ 0) = c_b,\quad c_R(z,\ 0) = 0$ $c_O(\infty,\ t) = c_b,\quad c_R(\infty,\ t) = 0$ $c_O(0,\ t) = e^{\eta} c_R(0,\ t)$ $D_O\left(\frac{\partial c_O}{\partial z}\right)_{z=0} = -D_R\left(\frac{\partial c_R}{\partial z}\right)_{z=0}$ with $\eta = \frac{F}{RT}(E - E^{o'})$ $\tau = (Da^2)^{1/2} t,\quad \zeta = z\left(\frac{a}{D}\right)^{1/3},\quad \theta_i = \frac{c_i}{c_b}$	$\theta_O(\zeta,\tau) = erf\left(\frac{\zeta}{2\sqrt{\tau}}\right) + erfc\left(\frac{\zeta}{2\sqrt{\tau}}\right)\left[\frac{\zeta^3}{24} + \frac{3k\tau\zeta^2}{8} + \frac{\tau\zeta}{4} - \frac{3k\tau\zeta^3}{8}\right] + \frac{e^{(-\zeta^2/4\tau)}\sqrt{\tau}}{\sqrt{\pi}}\left[\frac{\zeta^2}{4} - \frac{k\zeta^3}{4} - \frac{k\tau\zeta}{2}\right]$ $\theta_O(\zeta,\tau) + \theta_R(\zeta,\tau) = 1$ $\psi_{\lim}(\tau) = \frac{1}{\sqrt{\pi\tau}} + \frac{1}{4}\tau - \frac{k\tau^{3/2}}{2\sqrt{\pi}} + \frac{0.016666}{\sqrt{\pi}}\tau^{5/2} - \frac{0.09375}{6}k\tau^3 + \frac{1}{\sqrt{\pi}}0.0107142k^2\tau^{7/2}$ $\delta\left(\frac{a}{D}\right)^{1/3} = \frac{\theta_O(\xi \to \infty,\ \tau) - \theta_O(\xi \to 0,\ \tau)}{(\partial\theta_O/\partial\xi)_{\xi=0}} = [\psi_{\lim}(t)]^{-1}$ $\psi(\tau) = \frac{1}{1 + e^{\eta}}\psi_{\lim}(\tau)$ where $\tau = (Da^2)^{1/2} t,\quad \zeta = z\left(\frac{a}{D}\right)^{1/3},\quad \theta_i = \frac{c_i}{c_b},\quad k = 0.8175 \times Sc^{-1/3} = 0.8175 \times \left(\frac{\nu}{D}\right)^{-1/3}$

6. Conclusion

Most mathematical and theoretical models of rotating disc electrode are based on nonlinear reaction-diffusion differential equations. Various novel and advanced analytical methods such as the homotopy perturbation method, the Taylors series method, the Pade approximation technique, exp-function, hyperbolic function method, etc. have been employed to obtain approximate analytical solutions under steady and non-steady state conditions. Reliable analytical results are very useful for the analysis of various parameters like the thickness of the electrode, the loading of the different species, steady-state current, flux, diffusion rate, rate constant, reaction rate, the permeability of the porous medium, diffusion coefficients, kinematic viscosity, and voltammetry current is derived. In conclusion, rotating disc electrodes have made significant progress in power efficiency and stability since their conception. However, there is still a need for further theoretical and simulation research to make them a more technically and commercially feasible solution for wearable, implantable, and portable devices powering.

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